Theme 1, Week 7
Testing for mixed strategy equilibria in professional sports

EC340 Topics in Applied Economics (a)

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WARWICK

Structure of lectures

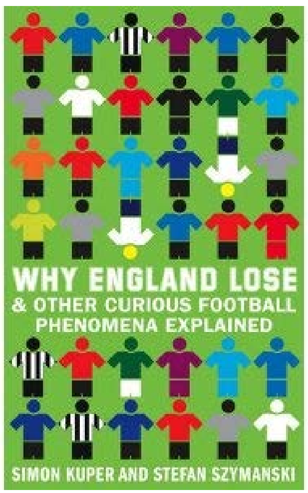
1. Sport and Game Theory
2. Theorem of Minimax
3. Matching Pennies
4. Penalties in Football
5. Testing for Mixed Strategy Nash Equilibria
6. Empirical research: experimental and data from professional sports
7. Other observations from research
8. Conclusions

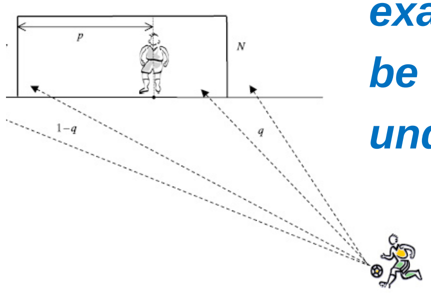
References

Some online resources

Sport and Game Theory

- The competitive *win or lose (zero sum)* nature of many professional sports may be close approximations to situations often studied in the theory of games
- Studying behaviour of professional sports players may present a way to test predictions emanating from game theory
- Alternative testing frameworks are laboratory run experiments and natural experiments conducted in the field
- In this theme we will be touching on some of these aspects





Penalties are often dismissed as a lottery; economists tell both the kicker and goalkeeper exactly what to do. And best of all, penalties may be the best way in the known world of understanding game theory.

Kuper & Szymanski (2009), Ch. 6, p. 126

2-player strictly competitive games

- Suppose you have no idea what your opponent will choose
  - You have never previously played with your opponent
  - Which way will the goalkeeper dive to save my penalty?
- Associate with each of your available actions the worst possible outcome for you – then select that action which has the maximum value among this 'worst' set (maximize the minimum)
  - This conservative behaviour is known as maximin – maximising the minimum payoff predicated on fearing the worst
- Example: Player 1 chooses either *T* or *B* and Player 2 chooses *L* or *R*

	L	*R*
T	*3*	*0*
B	*1*	*2*

In the payoff matrix are the payoffs for Player 1. What is Player 1's maximin strategy? Assume Player 2's payoffs are the negative values of Player 1

Minimax Theorem (von Neumann 1928)

- **Theorem:** For every two-person, zero-sum game with finitely many strategies, there exists a value V and a mixed strategy for each player, such that
  - (a) Given Player 2’s strategy, the best payoff possible for Player 1 is V, and
  - (b) Given Player 1's strategy, the best payoff possible for player 2 is $-V$
    - A constructive proof is available in Dantzig (1954)
- Matching Pennies is a classic example where a solution yields probabilistic strategy choices (*mixed strategies*)
  - Player 1 and 2 each has a penny and must secretly turn the penny to heads (H) or tails (T). The players reveal their choices simultaneously. If the pennies match Player 1 keeps both pennies. If the pennies do not match Player 2 keeps both pennies.

	H	*T*
H	*1*	*-1*
T	*-1*	*1*

In the payoff matrix are the payoffs for Player 1. Player 2’s payoffs are the negative values of Player 1. There is no Nash equilibrium in pure strategies.

Randomisation in Matching Pennies

- It is clear an equilibrium with pure strategies does not hold in the matching pennies game
- What if we regard players as mixing strategies?
  - Assigning probabilities to each of their pure strategies and delegating choice in the game to the outcome of a random draw from a distribution consistent with underlying probabilities assigned by each player to their respective pure strategies
- Player 1 considers his strategy assuming that Player 2 chooses *H* with probability q and *T* with probability $1-q$
  - If Player 1 chooses *H* her payoff is: $q \times (1) + (1-q) \times (-1) = 2q - 1$
  - If Player 1 chooses *T* her payoff is: $q \times (-1) + (1-q) \times (1) = 1 - 2q$
- Is there a Nash equilibrium where each player chooses a probability and given these probability choices, each player cannot achieve a higher expected payoff given the probability of the opponent?
  - In other words a mixed strategy equilibrium that constitutes best responses to one another's strategy choice, where the strategy choice is over mixing

Matching Pennies Nash equilibrium

- What is Player 1's best response if Player 2 chooses *q*?
- It cannot be the case Player 1 strictly prefers *H* or *T* – why?
- Therefore it follows for any Nash equilibrium it must be the case for Player 1:

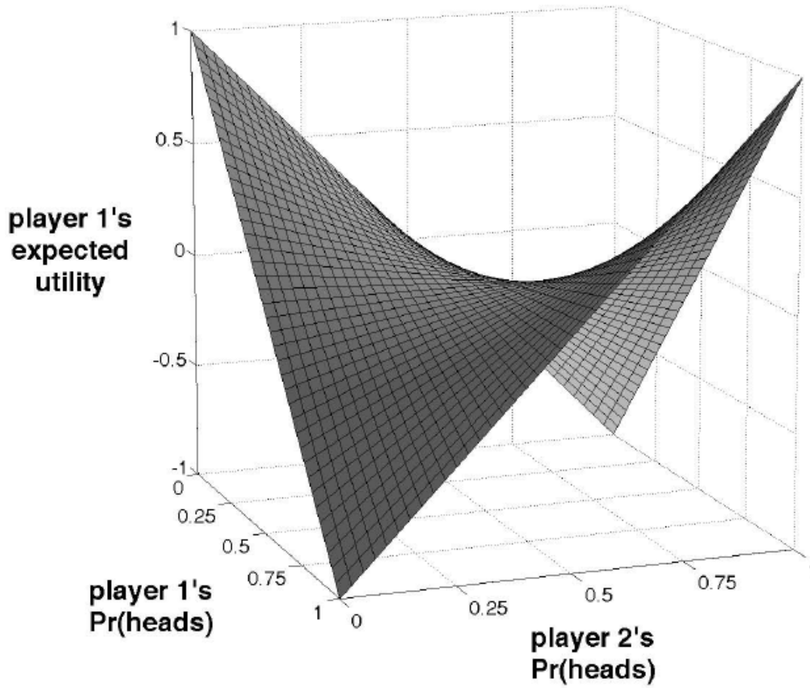
$$1-2q=2q-1 \quad \Rightarrow q=\frac{1}{2}$$

- By symmetry the same reasoning holds for Player 2, hence in any Nash equilibrium it must be the case that $p=1/2$
- Given the 2x2 system the Nash equilibrium is unique and involves each player choosing each strategy with a probability half
- The Nash equilibrium in this game is equivalent to that predicted under the Theorem of Minimax
  - All Minimax solutions in finite zero sum games comprise Nash equilibria, and the only Nash equilibria in these games are Minimax strategies

Matching Pennies: A useful metaphor for conflict in sport?

- Is the matching pennies type game useful for predicting highly competitive sport situations?
- Might a player in tennis randomise the location of a serve and a recipient of a serve decide to move left or right randomly?
- A penalty kicker in football may kick left or right randomly, and a goalkeeper may dive to the left or right randomly?
- Does it make sense to regard these actions (strategies) as if they are the outcome of a random device?
- Game theory predicts in situations where interests are diametrically opposed between two players and a pure strategy equilibrium does not exist that there will exist a mixed strategy Nash equilibrium
- How can this prediction be tested?
- Some researchers have turned to sports to analyse data to test whether players do accord with the theory of Maximin
  - In Matching Pennies the mixed strategy equilibrium is consistent with Maximin play

Expected utilities in Matching Pennies



Penalties in Football: Do they accord with predictions of game theory?

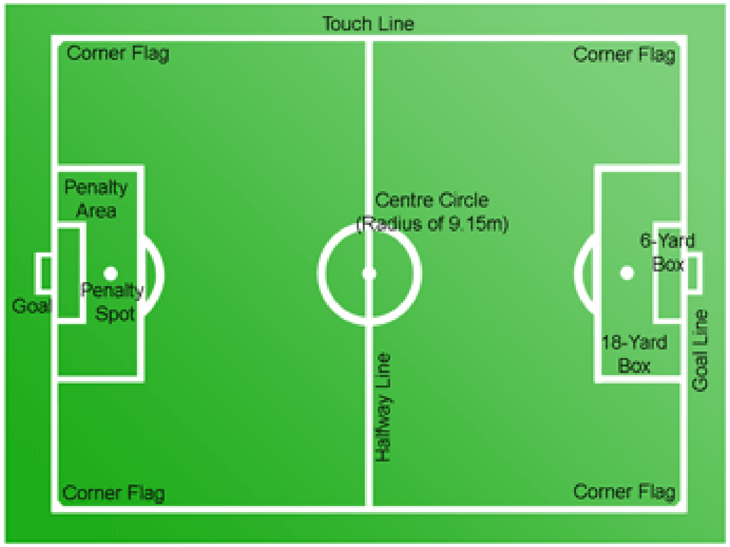
Whether that was a penalty or not, the referee thought otherwise
John Motson, legendary British TV and radio football commentator

- How to take penalties: *Freakonomics* explains 'You too can be a World Cup winner: just kick the ball straight'*
  - Why do Dubner and Levitt prescribe a straight kick?
  - If the goalkeeper adopts a similar strategy then presumably most kicks are saved! What do the data show?

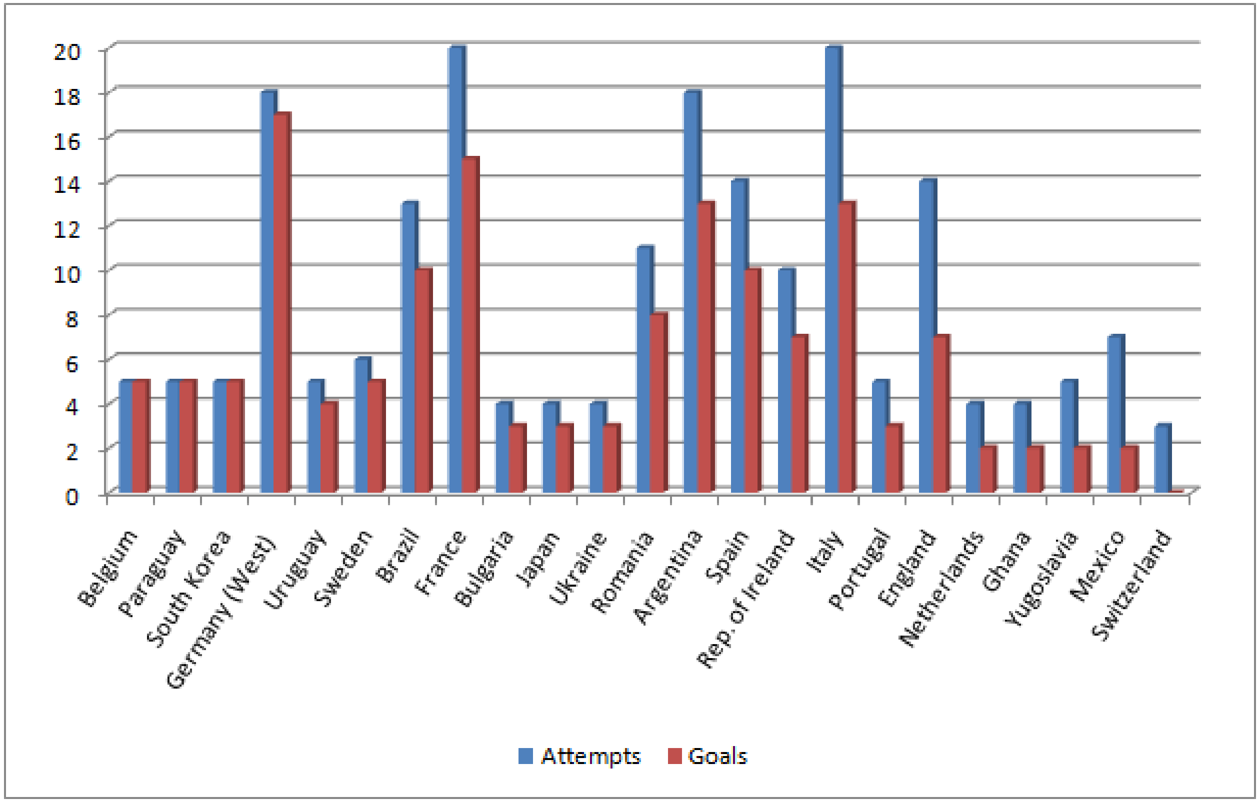
* Stephen J. Dubner and Steven D. Levitt, *The Times*,12 June 2010

The Penalty Kick in Football (Soccer): Rules

- Penalty kick taken from penalty spot, a spot midway between the goalposts some 12 yards (11 m) from the goal
- The penalty kick taker (**PK=Penalty Kicker**) must be clearly identified to the referee
- All players other than the defending goalkeeper (**GK=Goalkeeper**) and the PK must be outside the penalty area, behind the penalty mark, and at least ten yards (9.15 m) from the ball (i.e. outside the penalty arc) until the ball is kicked
- The GK must remain between the goalposts on the goal-line facing the ball until the ball is kicked, but may move from side to side along the goal-line. If the GK moves forward before the ball is kicked, then the penalty must be kicked again if a goal is not scored
- After the referee blows his whistle, which is the signal for the kick to be taken, the PK must kick the ball in a forward direction. The ball must be kicked after a run-up by the PK, who may slow his run but may not completely stop once the run-up has begun. If the PK scores after violating this rule, the kick must be re-taken
- Penalty introduced in England by the FA in 1891, on a line across the area, from 1902 it became a spot – first ever scored by Wolverhampton 14 September 1891
- FIFA approved penalty shoot outs in 1970, first shoot-out in 1982 World Cup



Some data on penalties: World Cup penalty shoot outs

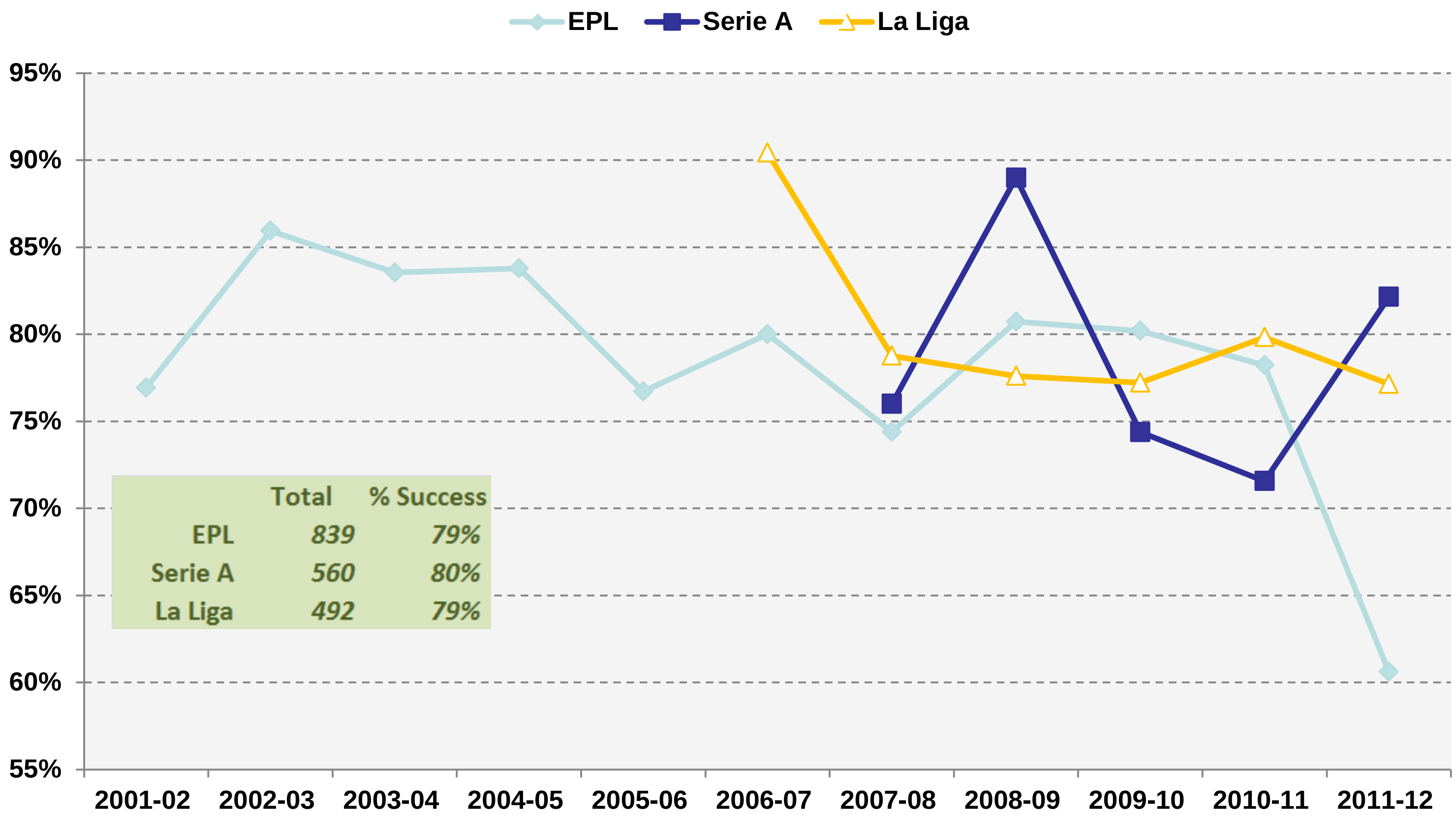


Up to 6 July 2010 (including Ghana *vs* Uruguay) there have been 204 penalties in shoot-outs, of which 144 were successful (71%)

71% success rate is slightly below the success rate for penalties within the course of normal play, which typically ranges from 75-85% (**Bar-Eli (2009)**)

In the 1986 Mexico World Cup about 20% of the 42 kicks shot landed in a 2m wide zone at the centre of the goal (**Bar-Eli and Friedman (1988)**)

Some data on penalties:
Normal play in EPL, Serie A and La Liga

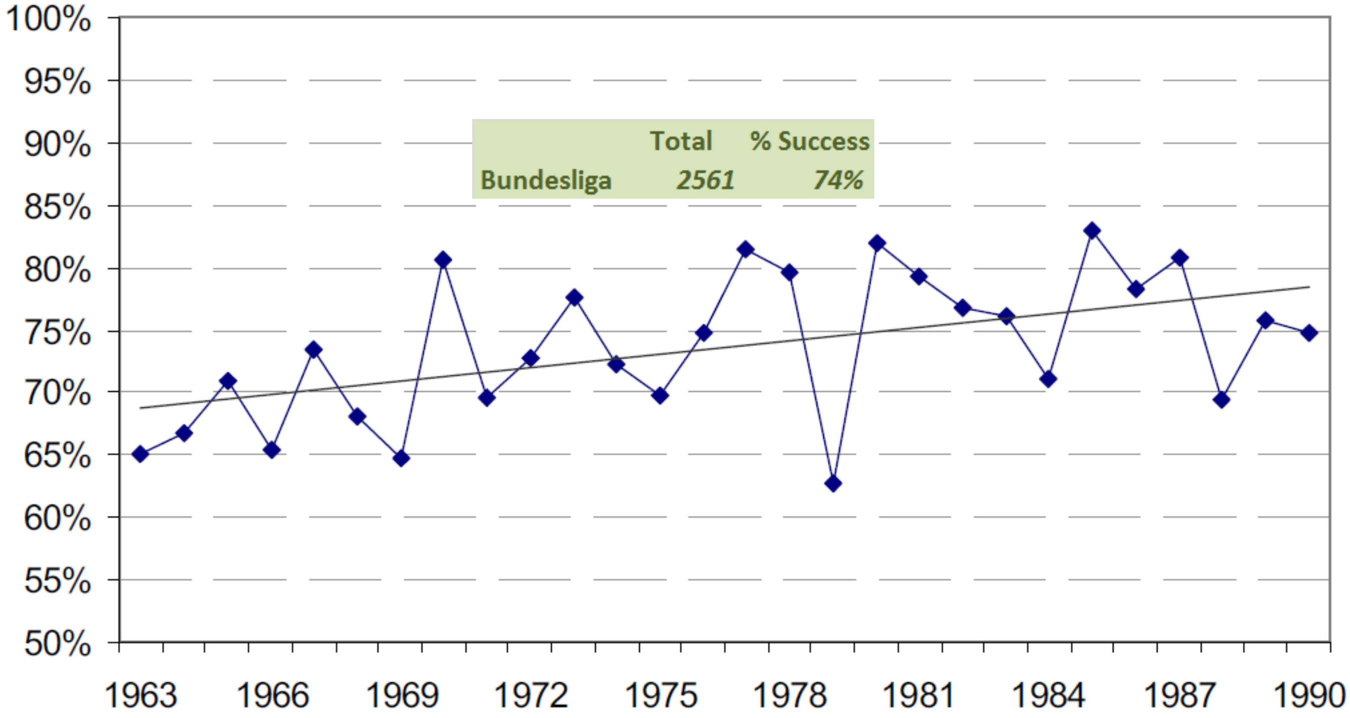


	Total	% Success
EPL	*839*	*79%*
Serie A	*560*	*80%*
La Liga	*492*	*79%*

Data sources: http://www.football-lineups.com and
http://www.myfootballfacts.com/Premier_League_Penalty_Statistics.html

Some data on penalties: Normal play in Bundesliga

Scoring rate over time



Source: Wolfgang Leininger and Axel Ockenfels (2007) "[The Penalty-Duel and Institutional Design: Is there a Neeskens-Effect?]" Ruhr Economic Papers #4

What do the data suggest?

- The average success rate is around 80%
  - On average a goalkeeper would be expected to save 1 in 5 penalties
- The data show asymmetry between the kicker and the goalkeeper
  - Symmetric Matching Pennies game is unlikely to be a useful metaphor, but closer inspection shows penalties in normal play to be in the Matching Pennies family (see Slide 23)
- The similarity in success rates across leagues and time suggest on average a common strategy is deployed in practice
- We need to consider further detail to understand how the penalty kick situation may reflect an underlying game and whether the participants accord with minimax strategies in practice
- Several economic papers have taken this route, which we discuss below
- Before looking at the academic research, we shall address coaching and other factors at influence

The Role of the Coach

David Moyes Manager Everton FC, *Sunday Times* 27 June 2010

In a good team you win together and you lose together, the penalty shootout is the loneliest experience in football and the trick is to make participants feel they are less on their own. The players taking spot-kicks need to know that responsibility does not weigh solely on their shoulders but is shared, and the same goes for your goalkeeper. As a manager, you can take pressure off your men by making yourself accountable for success and failure.

This means that when your players walk up to the spot they — and, more importantly, you — have decided exactly what they're going to do,. When we won against Manchester United, all my lads did exactly, from the spot, what we'd said they would beforehand. I feel you take pressure away from individuals that way. You say to a player: "All I want you to do is X." If it doesn't go in, then fine. We score together, we miss together when it is penalties.

Mentally, it's important to stay calm and ignore the goalkeeper. The keeper will most likely be jumping around trying to distract you. It is a good idea to make a quick check of the keeper's position just to make sure he is lined up properly, but other wise don't look at him. To enter a state of flow or 'being in the zone' when taking a penalty shot you need to <u>stop thought</u>. Sure you can have a pre-decided idea as to where you are going to blast the ball. <u>But thought or any self consciousness about what you are doing will just block your success.</u> In order to be able to reach this state consistently, you have to practice under pressure. Ultimatesoccercoaching.com

Coaching and options for the kicker

"A well-placed ball, high to the corner, will not be stopped by the goalkeeper even if he anticipates it" says Professor Tom Riley, Liverpool John Moores University. "There is not enough time to react, so a kick placed in this area would have a 100% strike rate." "Some players blast the ball straight down the middle, assuming that the goalkeeper will move, but it's not always successful."

Conventional wisdom says to go for the side netting (lower 90), low and just down inside the post. While this is an easier strike, a keeper that guesses correctly can get to the spot and make the save.

Source: http://www.ultimatesoccercoaching.com/soccer-kick/how-to-take-a-penalty-kick.html

Visual Search Behaviour

Savelsbergh et al. (2002) used a novel methodological approach to examine skill-based differences in anticipation and visual search behaviour during the penalty kick. Expert and novice goalkeepers were required to move a joystick in response to penalty kicks presented on film. The proportion of penalties saved was assessed, as well as the frequency and time of initiation of joystick corrections. Visual search behaviour was examined using an eye movement registration system. Expert goalkeepers were generally more accurate in predicting the direction of the penalty kick, waited longer before initiating a response and made fewer corrective movements with the joystick. The expert goalkeepers used a more efficient search strategy involving fewer fixations of longer duration to less disparate areas of the display. The novices spent longer fixating on the trunk, arms and hips, whereas the experts found the kicking leg, non-kicking leg and ball areas to be more informative, particularly as the moment of football contact approached. No differences in visual search behaviour were observed between successful and unsuccessful penalties. The results have implications for improving anticipation skill at penalty kicks.

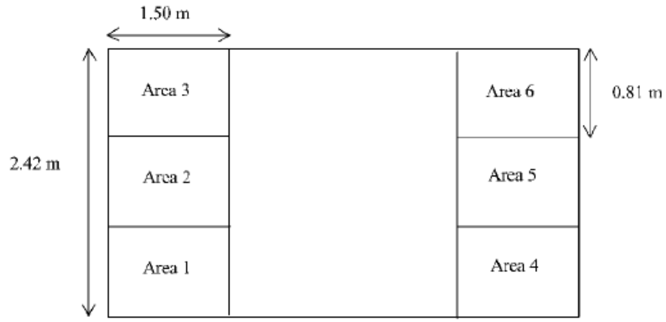


Fig. 1. The goal divided into six areas for placement of penalties and joystick movements.

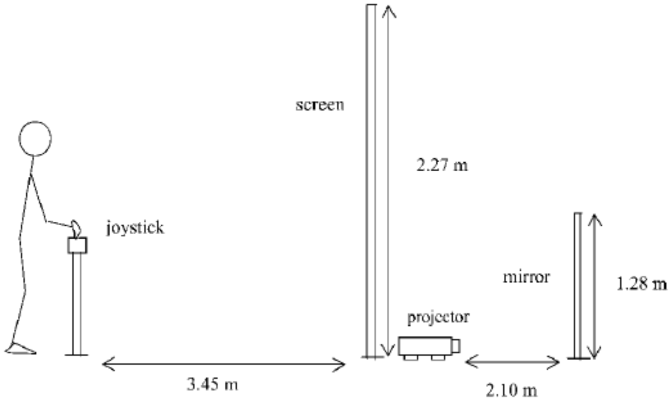
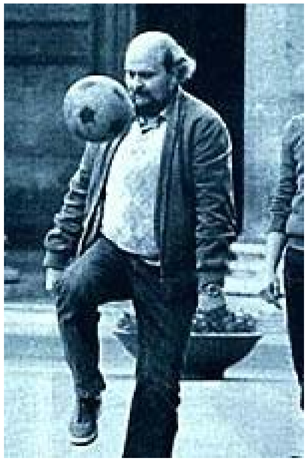
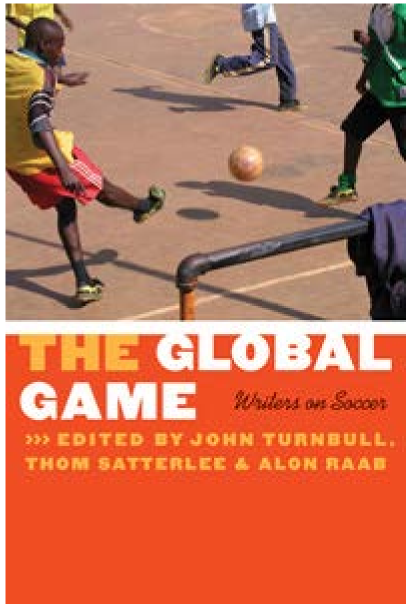
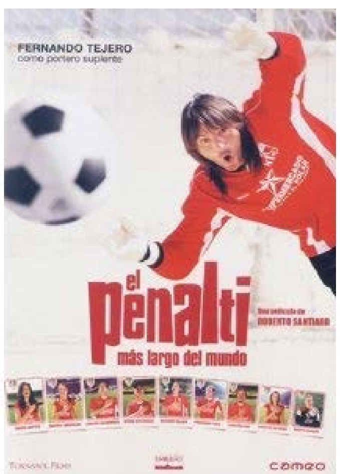


Fig. 2. A side view of the experimental set-up.

Level-k reasoning: Osvaldo Soriano
The Longest Penalty Ever







***Ch.* 18 in *The Global Game*, edited by John Turnbull *et al.* (2008)**

A short story by the late Argentine writer and journalist based on a real experience in the 1950s.

Story: A match in the Argentine provinces has to be abandoned seconds before the final whistle when a corrupt referee is laid unconscious by an angry player objecting to a penalty kick awarded to the opposition. The league decides that the last 20 seconds of the game comprising the penalty kick, shall be played one week later. Everyone has a week to prepare.

***Extract:* At a dinner a few nights before the penalty Gato DÍaz the GK discusses:**
“Constante kicks to the right’
‘Always’ said the president of the club.
‘But he knows that I know’
‘Then we’re fucked’
‘Yeah, but I know that he knows,’ said el Gato.
‘Then dive to the left and be ready,’ said someone at the table. ‘No. He knows that I know that he knows,’ said Gato DÍaz, and he got up to go to bed.

Palacios-Huerta (2003) “[a penalty] requires unpredictability and mutual outguessing’” (p. 396)

Toni Schumacher: learning and preparation

Schumacher kept a record of notes about opposition penalty kickers and their preferred side to kick, an excerpt is shown below:

Source: Wolfgang Leininger and Axel Ockenfels (2007)

Harald Anton Schumacher (born 6 March 1954 in Düren, West Germany), commonly known as Toni Schumacher, is a German former football goalkeeper, and a member of the West German national team. He won the 1980 European Championship and lost two World Cup finals, in 1982 and 1986. However, he is perhaps best remembered for a highly controversial incident in the 1982 FIFA World Cup semifinal against France when he collided with and seriously injured French defender Patrick Battiston.

End of Part 1: No pressure antics…

Jerzy Dudek, *AC Milan v Liverpool*, 2005, *Game: Champions League Final shootout*

Back to Slide 23

Testing for mixed strategies: experimental evidence

O'Neil (1987) conducted experiment with 25 pairs of subjects over 105 rounds (5 cents payoff), claims outcome supports minimax prediction:

		Player 2					
		Ace	*2*	*3*	*J*	MSNE	Empirical Frequencies
Player 1	*Ace*	-5	5	5	-5	0.20	0.221
	2	5	-5	5	-5	0.20	0.215
	3	5	5	-5	-5	0.20	0.203
	J	-5	-5	-5	5	0.40	0.362
	MSNE	0.20	0.20	0.20	0.40		
	Empirical Frequencies	0.226	0.179	0.169	0.426		

Brown and Rosenthal *Econometrica* (1990) re-examined the O'Neil game and rejected minimax play in both frequencies and serial independence, and observed an Ace bias

The theory's consistent failure in experimental tests (aside from O'Neill, 1987) raises the question whether there are any strategic situations in which people behave as the theory predicts – some believe professional sports offers such a situation

Back to Slide 15

Chiappori *et al*, *AER* (2002)

- Set out to test the empirical relevance of mixed strategies
- Noted that most tests previously had been laboratory based e.g. O'Neill (1987), see Slide 22
- First major contribution to test indirectly Mixed Strategy Equilibria (MSE) using penalty kicks (using a variety of econometric techniques)
  - Walker and Wooders (2001) test minimax looking at data from Wimbledon
- Assumes GK and PK can choose one of three strategies: *L*, *C* and *R* and that the strategies are chosen simultaneously (heterogeneity accounted for by assuming kicker's natural side is *L*, opposite side *R*) and $\pi_i > P_i$
- Data on penalty kicks (total in sample 459) from French and Italian leagues over a three year period from 1997-2000

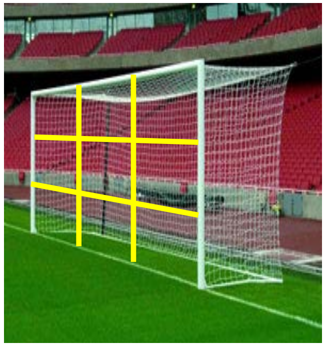
K_i \ G_i	L	C	R
L	P_L	π_L	π_L
C	μ	0	μ
R	π_R	π_R	P_R .

Table 1—Observed Scoring Probabilities, by Foot and Side

Kicker	Goalie: Correct side	Goalie: Middle or wrong side
Natural side ("left")	63.6 percent	94.4 percent
Opposite side ("right")	43.7 percent	89.3 percent

Chiappori *et al*, *AER* (2002)

- Assumptions are made with regard to the relative payoffs (probabilities of scoring) which are supported by the data
- A critical assumption of the model is that the goalie and the kicker play simultaneously. They cannot reject this assumption empirically.
- Given the assumptions the authors show there exists a unique mixed strategy equilibrium
- Data do not reject null hypothesis that scoring probabilities are equal for PKs across *R*, *L* and *C* given the assumption all goalies are identical
  - *This is the crucial test for indifference across the pure strategies*
- Test lends support to the proposition that professional players may be playing Minimax strategies



Ignacio Palacios-Huerta, *REStud* (2003)

- A test for von Neumann's Minimax Theorem in a 'natural' setting
  - 2 player one-shot zero sum game

	L	R
L	π_{LL}	π_{LR}
R	π_{RL}	π_{RR}

The probabilities of scoring represent the payoffs. The kicker seeks to maximize expected payoff, the goalkeeper seeks to minimize the expected payoff.

  - As long as $\pi_{LR} > \pi_{LL} < \pi_{RL}$

    $\pi_{RL} > \pi_{RR} < \pi_{LR}$

There exists a unique Nash equilibrium in mixed strategies

Palacios-Huerta

- The simple 2x2 zero sum game provides two testable hypotheses:
  1. The probability a goal is scored for the kicker should be the same for each of the kicker's strategies across the strategies of the goalkeeper

     Let g_L be the goalkeeper's probability of choosing L

     Given g_L the game predicts that the kicker's probability of scoring choosing L or R is the same, i.e. $p_L^k = p_R^k$ where:

$$p_L^k = g_L \pi_{LL} + (1 - g_L)\pi_{LR}$$

$$p_R^k = g_L \pi_{RL} + (1 - g_L)\pi_{RR}$$

     Let k_L be the kicker's probability of choosing L

     Given k_L the game predicts that the goalkeeper's probability of saving choosing L or R is the same, i.e. $p_L^g = p_R^g$ where:

$$p_L^g = k_L(1 - \pi_{LL}) + (1 - k_L)(1 - \pi_{RL})$$

$$p_R^g = k_L(1 - \pi_{LR}) + (1 - k_L)(1 - \pi_{RR})$$

Palacios-Huerta

2. The second testable hypothesis is that player's choices must be serially independent given constant payoffs across games
    - It might be argued that penalties in games towards the end of a season may be associated with different payoffs to those earlier in a season

- Data on 1,417 penalty kicks in professional soccer (Spain, Italy, England and few others), detailed actions and outcomes from September 1995 to June 2000

Palacios-Huerta (2003)

TABLE 1

Distribution of strategies and scoring rates

Score difference	#Obs.	LL	LC	LR	CL	CC	CR	RL	RC	RR	Scoring rate
0	580	16·9	1·3	21·0	4·3	0·8	5·6	19·4	0·6	27·9	81·9
1	235	19·1	0	19·1	4·2	0	2·5	28·0	0	26·8	77·8
−1	314	19·7	0·9	25·8	1·9	0	6·4	20·0	0·6	30·2	80·2
2	97	23·7	2·0	17·5	5·2	0	0	20·6	1·0	29·9	75·2
−2	114	26·3	0	25·4	3·5	0	3·5	16·6	0	24·5	78·0
3	27	14·8	0	18·5	3·7	0	11·1	22·2	0	29·6	77·7
−3	23	30·4	0	30·4	0	0	0	21·7	0	17·4	82·6
4	7	42·8	0	28·5	0	0	0	14·2	0	14·2	100
−4	12	25·0	0	25·0	0	0	16·6	16·6	0	16·6	83·3
Others	8	50·0	0	0	0	0	12·5	37·5	0	0	87·5
Penalties shot in:											
First half	558	21·1	0·8	19·8	3·9	0·3	3·5	20·0	0·3	29·7	82·9
Second half	859	18·7	0·9	23·2	3·3	0·3	3·6	22·8	0·5	26·3	78·3
Last 10 min	266	21·8	0	21·0	0·3	0	0·7	25·1	0	30·8	73·3
All penalties	1417	19·6	0·9	21·9	3·6	0·3	3·6	21·7	0·5	27·6	80·1
Scoring rate	80·1	55·2	100·0	94·2	94·1	50·0	82·3	96·4	100·0	71·1	

Note: The first letter of the strategy denotes the kicker's choice and the second the goalkeeper's choice. "R" denotes the R.H.S. of the goalkeeper, "L" denotes the L.H.S. of the goalkeeper, and "C" denotes centre.

Palacios-Huerta (2003)

TABLE 2

Distribution of strategies and scoring rates by kicker type

Score difference	#Obs.	LL	LC	LR	CL	CC	CR	RL	RC	RR	Scoring rate
		Left-footed kickers									
0	174	17·8	1·7	20·1	6·3	0	8·6	22·9	0·5	21·8	82·7
1	73	28·7	0	30·1	4·1	0	2·7	19·1	0	15·0	78·0
−1	92	29·3	1·0	26·0	1·0	0	2·0	21·7	1·0	18·4	82·6
2	29	51·7	0	13·7	3·0	0	0	10·3	0	20·6	72·4
−2	30	40·0	0	13·3	3·0	0	3·0	20·0	0	20·0	76·6
All penalties	406	29·3	1·4	20·4	4·4	0	3·9	23·8	0	16·5	
Scoring rate	81·0	62·1	100	95·1	94·4	0	81·2	93·8	0	61·2	
		Right-footed kickers									
0	406	16·4	1·2	21·4	3·4	1·2	4·4	20·4	0·7	30·5	83·2
1	162	14·8	0	14·2	4·3	0	2·4	32·1	0	32·1	77·7
−1	222	15·7	1·0	25·6	2·2	0	0	19·3	1·0	35·1	80·6
2	68	11·7	2·9	19·1	5·8	0	0	25·0	1·4	33·8	76·4
−2	84	21·4	0	29·7	3·5	0	3·5	15·4	0	26·2	78·5
All penalties	1011	15·8	0·6	22·5	3·2	0·5	3·4	20·8	0·6	32·1	
Scoring rate	79·8	50·0	100	93·8	93·9	60·0	82·8	97·6	100	73·2	

Note: The first letter of the strategy denotes the kicker's choice and the second the goalkeeper's choice. "R" denotes the R.H.S. of the goalkeeper, "L" denotes the L.H.S. of the goalkeeper, and "C" denotes centre.

Palacios-Huerta (2003)

- Penalty kickers in the data fall in one of two categories: left footed or right footed.
- A right footed "kicker's natural side" is to place the ball to the RHS of the GK; a left footed "kicker's natural side" is to place the ball to the LHS of the GK
- *L* is used to denote the kicker's non-natural side
- The probability of success matrix and the mixed strategy Nash equilibrium and actual frequencies (aggregate data) are derived as follows (he assumes that *C* is included on the natural side *R*):

	g_L	$1-g_L$
k_L	58·30	94·97
$1-k_L$	92·91	69·92

	g_L (%)	$1-g_L$ (%)	k_L (%)	$1-k_L$ (%)
Nash predicted frequencies	41·99	58·01	38·54	61·46
Actual frequencies	42·31	57·69	39·98	60·02

Palacios-Huerta (2003)

Results show:

1. Winning probabilities statistically identical across strategies
   - Experimental data have found this difficult to obtain
2. Players' choices serially independent (i.i.d.)
   - Experimental data in psychology and economics tends to find players 'switch strategies' too often to be consistent with random play (also found in Walker and Wooders with professional tennis players first serve – which were negatively serially correlated)
3. Claims this is the first empirical support for von Neumann's theorem in a natural setting

Professionals *vs* Students

Palacios-Huerta & Volij (PHV) *Econometrica* (2008)

- Professional footballers placed in the laboratory to play a game modelled on the penalty kick and their strategies are shown to conform to the minimax equilibrium and exhibit serial independence, whereas students (non-economists and non-mathematicians) perform significantly differently from the equilibrium and strategies exhibit negative serial correlation (are not random over repetitions). The subjects also played the less familiar O'Neill game and professional footballers again acted in accordance with the minimax equilibrium, whereas students did not.

Wooders, John *Econometrica* (2010)

- Re-examined the PHV data, showing that, in fact, the play of professionals is inconsistent with the minimax hypothesis in several important respects: including negatively correlated strategies between the first half of the data and the second half. He also shows the behaviour of students conforms more closely to the minimax hypothesis. Levitt *et al* (2010) Econometrica also show similar results for US professional soccer players and poker players.

Mixed Up about Strategies?

- With apologies to Reinhard Selten (1975) are penalty kicks a case of a *trembling foot perfect equilibrium*!
- The PK chooses a strategy aiming for a zone, there is a probability of error in executing the action (alternatively, the strategy realised may vary from the strategy planned – *the trembling foot*)
- GK does not make an error, PK has probability of error
- If the ball hits the target at any zone with equal probability, the goalkeeper may as well dive randomly or not dive at all
- Obviously, the GK seeks to signal a weak side (separating equilibrium) to try and influence the PK's choice and then defends that side
- I am alluding to *deliberate randomisation* ***e.g.*** Reny and Robson *GEB* (2004)
- Alternatively could be a *Quantal Response Equilibrium* (McKelvey and Palfrey (1995)), where an error in choosing a pure strategy is interpreted as bounded rationality – difficult to view players in penalty sub-game as consistent with bounded rationality
- Another possibility is an application of *Level-k reasoning* (Nagel (1995); Stahl and Wilson (1995); Costa-Gomes and Crawford (2006)). While penalty kicks involve guessing, the game structure is not one where iterative dominance is apparent - but level-k reasoning could apply
- Problem of identification: can we distinguish randomisation from heterogeneous play?
- *Ambiguity aversion*? Players are uncomfortable with unknowns and behave in ways contrary to expected utility theory – randomisation is a hedge against making a poor choice

Mixed Up about Strategies? Do players guess?

- Ariel Rubinstein (1994) “[Matching Pennies] is classically used to motivate the notion of mixed strategy equilibrium, but randomization is a bizarre description of a player’s deliberate strategy in the game. A player’s action is a response to his guess about the other player’s choice; guessing is a psychological operation that is very much deliberate and not random” (Osborne and Rubinstein (1994), p.37)
- Eliaz and Rubinstein (2011) (*Games and Economic Behavior*) Examine framing effects (labelling of players and actions, sequencing of moves) in a finitely repeated matching pennies game motivated by a game of marbles described in Edgar Allan Poe’s short story “The Purloined Letter” – where one boy chooses the number of marbles (1 or 2), and the second guesses the choice – *this is much like the PK choosing where to kick and the GK guessing where he is kicking.*
- Timothy Dang (2009) (U Arizona) looks at a game of matching pennies with guessers and players (the guessers mirror the players using *z-Tree)* to see whether different players play different pure strategies and his results appear to show ambiguity aversion is the majority motivation for randomization

Conclusion

- Play in professional sports may be a natural way to test for mixed strategy Nash equilibria
- The penalty kick in football has been viewed as a two-person one-shot simultaneous move zero sum game
- This interpretation makes it analogous to a *matching pennies* game
- Analyses of penalty kick data appear to show that professionals play mixed strategies, as the matching pennies game predicts
- Doubtful whether the PKs at the moment of a kick randomize (some may do occasionally), though GKs may ‘guess’
- Preparation involves choosing a pure action in an informed fashion (using history where available a la Schumacher) and adding randomness to avoid prior detection (ambiguity aversion)
- Heterogeneous players and settings make field data potentially unreliable, and the penalty is a subset of a larger game which is subject to external review
- A very interesting line of research that has developed over the last 10 years and presented some much needed progress on understanding mixed strategies

References

- **Azar, Ofer H. and Bar-Eli, Michael (2010)** "Do soccer players play the mixed-strategy Nash equilibrium?", forthcoming *Applied Economics*.
- **Bar-Eli M., and Azar O.H. (2009)** "Penalty kicks in soccer: an empirical analysis of shooting strategies and goalkeepers preferences" *Soccer & Society*, 10(2),183-91.
- **Bar-Eli, M., O.H. Azar, I. Ritov, Y. Keidar-Levin, and G. Schein. (2007)** "Action Bias Among Elite Soccer Goalkeepers: The Case of Penalty Kicks". *Journal of Economic Psychology* 28(5), 606-21.
- **Bar-Eli, M., and Z. Friedman (1988)** "Psychological Stress in Soccer: The Case of Penalty Kicks". *Soccer Journal* 33(6), 49–52.
- **Brown, James N and Rosenthal, Robert W, (1990)** "Testing the Minimax Hypothesis: A Re-examination of O'Neill's Game Experiment," *Econometrica*, 58(5), 1065-81.
- **Camerer, Colin F. (2003)** *Behavioral Game Theory: Experiments in Strategic Interaction*, Princeton University Press.
- **Chiappori, Pierre-Andre, Levitt, Steve and Timothy Groseclose (2002)** "Testing Mixed-Strategy Equilibria When Players Are Heterogeneous: The Case of Penalty Kicks in Soccer," *American Economic Review*, 92(4), 1138–51.
- **Coloma, Germán (2007)** "Penalty Kicks in Soccer: An alternative methodology for testing mixed-strategy equilibria," *Journal of Sports Economics*, 8(5), 530-45.
- **Costa-Gomes, Miguel A. and Crawford, Vincent P. (2006)** "Cognition and Behavior in Two-Person Guessing Games: An Experimental Study", *American Economic Review*, 96(5): 1737–68.
- **Dang, Timothy (2009)** "Gaming or Guessing: Mixing and best-responding in *Matching Pennies*" University of Arizona working paper.

References *cont.*

- **Eliaz, Kfir and Ariel Rubinstein (2010)** “Edgar Allan Poe's Riddle: Do Guessers Outperform Misleaders in a Repeated Matching Pennies Game?” forthcoming *Games and Economic Behavior.*
- **Gibbons, Robert (1992)** *A Primer in Game Theory* Harvester Wheatsheaf.
- **Goeree, Jacob K. and Charles A. Holt (2001)** “Ten Little Treasures of Game Theory and Ten Intuitive Contradictions”, *American Economic Review*, 91(5): 1402-22.
- **Harsanyi, J.C., (1973).** “Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points,” *International Journal of Game Theory,* 2, 1-23.
- **Kuper, Simon and Stefan Szymanski (2009),** *Why England Lose: And other curious phenomena explained*, HarperSport
- **McKelvey, Richard and Palfrey, Thomas (1995)** “Quantal Response Equilibria for Normal Form Games”, *Games and Economic Behavior* 10: 6–38.
- **Nagel, Rosemarie, (1995)** “Unraveling in Guessing Games: An Experimental Study” *American Economic Review*, 85(5): 1313–26.
- **O’Neill, B. (1987)** “Nonmetric Test of the Minimax Theory of Two-Person Zero-Sum Games,” *Proceedings of the National Academy of Sciences* 84, 2106-09.
- **Osborne, Martin J. and Ariel Rubinstein (1994)** *A Course in Game Theory*, MIT Press.
- **Ignacio Palacios-Huerta (2003)** “Professionals Play Minimax,” *Review of Economic Studies***,** 70(2), 395-415.
- **Palacios-Huerta, Ignacio and Oscar Volij, (2008)** “*Experientia Docet*: Professionals Play Minimax in Laboratory Experiments,” *Econometrica*, 76(1), 71-115.

References *cont.*

- **Reny, Philip J. and Arthur J. Robson (2004)** "Reinterpreting Mixed Strategy Equilibria: A Unification of the Classical and Bayesian Views," *Games and Economic Behavior.*48(2), 355-84.
- **Savelsbergh, Geert, J.P., Williams, A. Mark, van der Kamp, John and Paul Ward (2002)** "Visual search, anticipation and expertise in soccer goalkeepers" *Journal of Sports Sciences*, 20, 279-87.
- **Selten, Reinhard (1975)** "Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory*, 4, 25-55.
- **Stahl, Dale O. II, and Paul W. Wilson. (1995)** "On Players' Models of Other Players: Theory and Experimental Evidence." *Games and Economic Behavior*, 10(1): 218–54.
- **Turnbull, John C., Raab, Alon and Satterlee, Thom (2008)***The Global Game: Writers on Soccer*, Bison Books.
- **Von Neumann, John and Oscar Morgenstern (1944)** *Theory of Games and Economic Behavior*, Princeton, NJ: Princeton University Press
- **Walker, Mark and John Wooders (2001)** "Minimax Play at Wimbledon," *American Economic Review,* 91, 1521-1538.
- **Wooders, John (2010)** "Does Experience Teach? Professionals and Minimax Play in the Lab", *Econometrica*, 78(3), 1143–54.

Some online resources

- http://www.scienceofsocceronline.com/2009/04/penalty-kicks-by-numbers.html
  Summarises some research papers
- http://www.opposingviews.com/i/understanding-the-statistics-behind-world-cup-penalty-kicks
  Looks at Bar-Eli research
- Statistics on penalties: http://www.rsssf.com/miscellaneous/penalties.html
- On the Longest Penalty Ever: http://www.buchmesse.de/en/blog/argentina/2010/05/11/the-true-story-of-the-world%E2%80%99s-longest-penalty-kick-a-look-at-osvaldo-soriano-and-his-famous-short-story-but-in-real-life-the-better-team-won/